Nonlinear dynamics and performance enhancement of a flow energy harvester with time-delayed modulation

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Article Info

Article history:

Received Feb 5, 2026
Revised Mar 31, 2026
Accepted May 12, 2026

Keywords:

Coupling
Energy harvesting
Galloping-induced excitation
Nonlinear delayed oscillator
Piezoelectric electromechanical

ABSTRACT

This study investigates a nonlinear vibration-based energy harvesting (EH) system consisting of a galloping-induced oscillator coupled to an electrical circuit through piezoelectric transduction. The mechanical subsystem incorporates time-delayed feedback whose amplitude is periodically modulated at a frequency close to twice the natural frequency of the structure. Using the method of multiple scales, approximate analytical expressions are derived for the steady-state oscillation amplitude and the corresponding harvested electrical power. The results demonstrate that the proposed delay modulation mechanism can significantly enhance vibration amplitudes and EH efficiency within a specific range of wind velocities. This improvement is attributed to a delay-induced parametric resonance effect that amplifies the system response. The analytical predictions are validated through numerical simulations, showing excellent agreement between both approaches.

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1. INTRODUCTION

Vibration-based energy harvesting (EH) in systems incorporating delayed oscillators with time-periodic delay amplitude has been extensively explored in recent literature. Belhaq and Hamdi [1], it was demonstrated that a delayed Van der Pol oscillator with a modulated delay amplitude, coupled to an electromagnetic transducer, can generate quasi-periodic (QP) oscillations capable of harvesting energy efficiently over a broad parameter range. In this configuration, the delay term was applied within the mechanical subsystem. The scenario where delay effects are simultaneously introduced in both the mechanical and electrical domains was later examined in [2].

More recently, studies on a delayed Duffing oscillator have revealed that modulating the delay amplitude near the delay-induced parametric resonance leads to large-amplitude QP responses, which improve EH performance compared with purely periodic responses [3]. However, in certain harvester configurations subjected to aerodynamic or base excitations, QP vibrations were found to considerably reduce the harvested power beyond the flutter threshold, suggesting that operation in the QP regime should be avoided in such cases [4], [5].

Nevertheless, subsequent investigations highlighted that when time delay is properly incorporated, QP oscillations can instead be advantageous for energy extraction [6]–[8]. Ghouli *et al*. [6], for instance, a Duffing-type monostable harvester under harmonic excitation and coupled to a piezoelectric circuit was analyzed; the induced large-amplitude QP motions were shown to enhance harvesting efficiency. Appropriate tuning of the delay parameters allowed energy extraction over a broad excitation frequency range, far from resonance, while avoiding bistability and hysteresis phenomena.

Similarly, Belhaq *et al.* [7] explored QP vibration-based harvesting in a delayed nonlinear micro-electromechanical systems (MEMS) device modeled by a delayed Mathieu–Van der Pol–Duffing oscillator coupled to a delayed piezoelectric element. The results demonstrated superior harvesting performance compared to periodic regimes. More recently, Ghouli *et al.* [8] examined a forced Van der Pol oscillator coupled with a delayed piezoelectric circuit, revealing that an optimal delay frequency exists, maximizing both the QP amplitude and the harvested power.

Building on these previous findings [1]–[3], [6]–[8], the present work focuses on periodic vibration-based EH in a nonlinear delayed electromechanical harvester exposed to cross-flow excitation. The system features a modulated delay amplitude in the mechanical part and is coupled to an electrical circuit through a piezoelectric transducer. This configuration is relevant to applications where inherent delay effects arise from the mechanical attachment of the harvester [9]–[11], and thus, the delay cannot be regarded as an additional external input.

Vibration-based EH systems have gained significant attention due to their potential applications in autonomous sensors, structural health monitoring, and low-power wireless devices. In particular, flow-induced vibration harvesters offer a promising route for converting ambient wind energy into usable electrical power. Improving their efficiency and operational bandwidth remains a key challenge, especially under varying flow conditions. In this context, nonlinear control strategies such as delay modulation provide a powerful tool to enhance energy conversion performance.

Despite the significant progress achieved in the field of nonlinear EH with time-delayed systems, several limitations remain. Most existing studies focus on constant time delay or consider delay effects either in the mechanical or electrical subsystems separately. Furthermore, the role of periodically modulated delay amplitude in flow-induced vibration systems, particularly under galloping excitation, has not been sufficiently explored. In addition, the mechanisms through which delay modulation can enhance periodic EH performance via parametric resonance remain poorly understood.

Although significant progress has been made in nonlinear EH systems with delayed feedback, most existing studies remain limited to constant time-delay formulations or single-domain coupling configurations. In addition, relatively few works have investigated the role of periodically modulated delay in flow-induced vibration systems. The majority of previous research focuses on either Duffing-type or Van der Pol-type oscillators under harmonic or stochastic excitation, while the interaction between galloping dynamics and delay-induced parametric excitation has not been sufficiently explored. Moreover, only a limited number of studies provide a unified analytical and numerical framework for describing the influence of delay modulation on EH performance. These limitations highlight the need for further investigation into more general delayed nonlinear models capable of capturing richer dynamical behaviors.

This study aims to address these gaps by investigating a nonlinear electromechanical energy harvester with a periodically modulated time delay in the mechanical subsystem. The main contributions of this work are as follows: i) the development of an analytical model describing delay-induced parametric resonance in a galloping-based harvester, ii) the derivation of approximate solutions using the method of multiple scales, and iii) the demonstration that delay modulation can significantly enhance vibration amplitudes and harvested power over a specific range of flow velocities.

Although the present study focuses on galloping-induced flow excitation, the proposed modeling framework is not limited to this specific aerodynamic mechanism. The formulation of a nonlinear oscillator with time-delayed modulation and piezoelectric coupling can be extended to other types of flow-induced vibrations, such as vortex-induced vibrations (VIV) and aeroelastic flutter. Therefore, the present approach provides a general theoretical framework for analyzing delayed nonlinear EH systems under different flow conditions.

Compared with previous studies on delayed nonlinear EH systems, the present work introduces a periodically modulated delay amplitude acting as a parametric excitation in a galloping-based electromechanical oscillator. Most existing studies consider either constant delay or delay effects confined to a single subsystem, whereas the present model incorporates delay modulation in the mechanical feedback path, leading to a richer nonlinear dynamic behavior. This allows the emergence of delay-induced resonance phenomena that enhance both vibration amplitude and harvested power over specific operating conditions.

The main novelty of this work lies in introducing a periodically modulated delay amplitude in a galloping-based piezoelectric EH system and analytically characterizing its influence near delay-induced parametric resonance. Unlike previous studies that mainly consider constant delay or weakly time-varying delay effects, the present work demonstrates how controlled delay modulation can actively enhance nonlinear oscillations and improve EH performance. This approach reveals a new mechanism of delay-induced parametric amplification, where the interaction between aerodynamic galloping and modulated feedback delay leads to significant enhancement of vibration amplitude and harvested power within a specific operating range.

The remainder of this paper is organized as follows: section 2 presents the system model and derives analytical approximations for the periodic response and harvested power near the delay-induced parametric resonance using the multiple scales method. Section 3 investigates the influence of time-periodic delay amplitude in the mechanical subsystem on the overall EH performance. The concluding section (section 4) summarizes the main findings and discusses potential implications for future applications.

2. MODEL DESCRIPTION AND PERIODIC ENERGY HARVESTING

We consider a nonlinear electromechanical oscillator in cross flow as shown in the schematic presented in Figure 1. The mechanical oscillator consists of a bluff body of mass m coupled to an electrical circuit through a piezoelectric device. We assume that the mechanical and electrical components of the harvester are both under time delayed feedback such that the dimensionless governing equations for the system can be written as (1) and (2):

$$\ddot{x}(t) + \mu x(t) + \xi_1 \dot{x}(t) + \xi_3 \dot{x}(t)^3 + \gamma x(t)^3 - \kappa v(t) = \lambda(t) x(t-\tau) \quad (1)$$

$$\dot{v}(t) + \alpha v(t) + \dot{x}(t) = 0 \quad (2)$$

where $x(t)$ is the transverse displacement of the mass m and $v(t)$ is the voltage across the load resistance. The coefficients ξ_1 and ξ_3 are, the mechanical damping components, μ the naturel frequency of the harvester, γ the stiffness, κ the piezoelectric coupling term in the mechanical attachment, and α the reciprocal of the time constant of the electrical circuit. The parameter $\lambda(t)$ is the feedback gains in the mechanical attachment and τ is the time delay. We assume that the delay amplitude $\lambda(t)$ is modulated around a mean value as (3):

$$\lambda(t) = \lambda_1 \cos(\omega t) \quad (3)$$

where λ_1 and ω are, respectively, the amplitude and the frequency of the modulation. The coefficients ξ_1 and ξ_3 in (1) are defined as (4):

$$\xi_1 = 2[\xi_m - m a_1 U], \quad \xi_3 = \frac{2 m a_3}{U} \quad (4)$$

where ξ_m is the mechanical damping ratio, m is the flow to harvester mass ratio, U is the reduced wind speed, and the coefficients a_1 and a_3 account for the different geometries and aspect ratios of the bluff body. Note that the case of undelayed system ($\lambda_1 = \tau = 0$) was studied in [12].

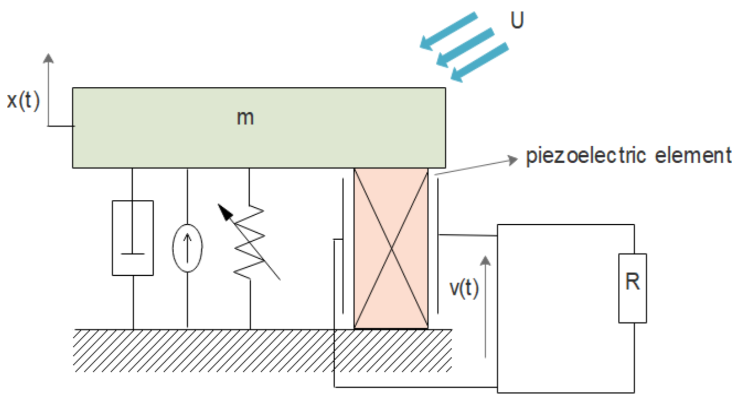


Figure 1. Schematic of the EH system

All variables and parameters used in the governing equations are defined immediately after their introduction to ensure clarity. The physical meaning of each coefficient is explicitly provided, including damping, nonlinear stiffness, piezoelectric coupling, and delay modulation parameters. The stability conditions derived in the analytical section are also interpreted in terms of physical bifurcation behavior and energy transfer mechanisms, allowing a clearer connection between mathematical results and physical interpretation.

The proposed model is based on the assumption of weakly nonlinear oscillations under steady cross-flow conditions. The aerodynamic loading is represented using a nonlinear galloping formulation, while the electrical subsystem is modeled as a linear resistive circuit. The time-delay term is interpreted as an effective memory effect resulting from fluid–structure interaction and electromechanical feedback mechanisms. Each parameter has a clear physical meaning: the mechanical damping terms control energy dissipation, the nonlinear stiffness governs large-amplitude oscillations, the piezoelectric coupling coefficient regulates energy transfer between the mechanical and electrical domains, and the delay modulation parameters define the strength and temporal characteristics of the feedback-induced parametric excitation. These assumptions allow capturing the essential nonlinear dynamics while maintaining analytical tractability.

In practical flow-induced EH systems, time delay may arise from several physical mechanisms. For instance, in galloping-based structures, the aerodynamic forces acting on the bluff body do not respond instantaneously to its motion, leading to a delay due to fluid–structure interaction effects. Additionally, structural flexibility and mechanical transmission elements may introduce delayed feedback in the restoring forces.

In electromechanical systems, delay can also originate from the piezoelectric coupling and the associated electrical circuit, where signal processing, sensing, and actuation may introduce finite response times. Therefore, the time delay considered in this model represents a generalized memory effect that accounts for the lag between the system response and the feedback forces.

To study the response of the harvester near the delay parametric resonance, we assume the resonance condition $\omega = 2\sqrt{\mu} + \sigma$ where σ is a detuning parameter. Approximation of solutions are obtained using the method of multiple scales [13] by introducing a bookkeeping parameter ϵ and scaling as $\xi_1 = \epsilon\tilde{\xi}_1,\ \xi_3 = \epsilon\tilde{\xi}_3,\ \gamma = \epsilon\tilde{\gamma},\ \kappa = \epsilon\tilde{\kappa},\ \lambda_1 = \epsilon\tilde{\lambda}_1,\ \sigma = \epsilon\tilde{\sigma}$. In (1) and (2) can be rewritten as (5) and (6):

$$\ddot{x}(t) + \mu x(t) = \epsilon[-\tilde{\xi}_1\dot{x} - \tilde{\xi}_3\dot{x}(t)^3 - \tilde{\gamma}x(t)^3 + \tilde{\kappa}v(t) + \tilde{\lambda}_1\cos(\omega t)x(t-\tau)] \tag{5}$$

$$\dot{v}(t) + \alpha v(t) + \dot{x}(t) = 0 \tag{6}$$

The method of multiple scales is employed under the assumption of weak nonlinearity and small perturbations in the system. The bookkeeping parameter $\epsilon(\epsilon \ll 1)$ is introduced to characterize the small magnitude of damping, nonlinear terms, and delay modulation amplitude. This allows the separation of time scales and the derivation of approximate analytical solutions near resonance conditions.

The validity of this perturbation approach is restricted to parameter regimes where nonlinearities, damping, and delay effects remain sufficiently small. In particular, the analytical results are expected to be accurate in the vicinity of the parametric resonance and for moderate values of the delay and its modulation amplitude.

It should be noted that for strongly nonlinear regimes or large delay values, higher-order effects may become significant, and the multiple scales approximation may lose accuracy. In such cases, numerical simulations are required to capture the full system dynamics. We seek a solution of (5) and (6) in the form as (7) and (8):

$$x(t) = x_0(T_0, T_1) + \epsilon x_1(T_0, T_1) + O(\epsilon^2) \tag{7}$$

$$v(t) = v_0(T_0, T_1) + \epsilon v_1(T_0, T_1) + O(\epsilon^2) \tag{8}$$

where $T_0 = t$ and $T_1 = \epsilon t$. Using the time derivatives $\frac{d}{dt} = D_0 + \epsilon D_1 + O(\epsilon^2)$, and $\frac{d^2}{dt^2} = D_0^2 + \epsilon^2 D_1^2 + 2\epsilon D_0 D_1 + O(\epsilon^2)$, where $D_i^j = \frac{\partial^j}{\partial^j T_i}$, substituting (7) and (8) into (5) and (6), and balancing terms of like powers of ϵ, we obtain up to the second order as (9) and (10):

$$D_0^2 x_0 + \mu x_0 = 0 \tag{9}$$

$$D_0 v_0 + \alpha v_0 + D_0 x_0 = 0 \tag{10}$$

and

$$D_0^2 x_1 + \mu x_1 = -2D_0 D_1 x_0 + \tilde{\kappa}v_0 - \tilde{\gamma}x_0^3 - \tilde{\xi}_3(D_0 x_0)^3 - \tilde{\xi}_1(D_0 x_0) + \tilde{\lambda}_1\cos(2\sqrt{\mu}T_0 + \tilde{\sigma}T_1)x_{0\tau} \tag{11}$$

$$D_0v_1 + \alpha v_1 = -D_1v_0 - D_0x_1 - D_1x_0 \tag{12}$$

The first-order solution is given by:

$$x_0(T_0, T_1) = A(T_1)e^{i\sqrt{\mu}T_0} + \bar{A}(T_1)e^{-i\sqrt{\mu}T_0} \tag{13}$$

$$v_0(T_0, T_1) = \frac{-i\sqrt{\mu}A(T_1)}{\alpha+i\sqrt{\mu}}e^{i\sqrt{\mu}T_0} + \frac{i\sqrt{\mu}\bar{A}(T_1)}{\alpha-i\sqrt{\mu}}e^{-i\sqrt{\mu}T_0} \tag{14}$$

where $A(T_1)$ and $\bar{A}(T_1)$ are unknown complex conjugate functions. Substituting (13) and (14) into (11) and (12), and eliminating the secular terms, yields the following (15):

$$-2i\sqrt{\mu}(D_1A) - 3\tilde{\gamma}A^2\bar{A} + 3\tilde{\xi}_3(i\sqrt{\mu})^3A^2\bar{A} - i\sqrt{\mu}\tilde{\xi}_1A - \frac{\tilde{\kappa}i\sqrt{\mu}A}{\alpha+i\sqrt{\mu}} + \frac{\tilde{\lambda}_1\bar{A}}{2}e^{i\sqrt{\mu}\tau}e^{i\tilde{\sigma}T_1} = 0 \tag{15}$$

assuming $A = \frac{1}{2}ae^{i\theta}$, where a and θ are the amplitude and the phase variables, we obtain the modulation as (16):

$$\begin{cases} \frac{da}{dt} = S_1a + S_2a^3 + S_5a\cos(\varphi) + S_6a\sin(\varphi) \\ a\frac{d\varphi}{dt} = S_3a + S_4a^3 + 2S_6a\cos(\varphi) - 2S_5a\sin(\varphi) \end{cases} \tag{16}$$

where $\varphi = \tilde{\sigma}T_1 - 2\theta$ and the coefficients $S_i(i = 1,\ldots,6)$ are given as. The solution in (13) and (14) reads as (17):

$$S_1 = -\frac{\xi_1}{2} - \frac{\kappa\alpha}{2\alpha^2+2\mu}, \quad S_2 = -\frac{3\xi_3}{8}, \quad S_4 = -\frac{3\gamma}{4\sqrt{\mu}}$$

$$S_3 = \sigma - \frac{\kappa\sqrt{\mu}}{\alpha^2+\mu}, \quad S_5 = \frac{\lambda_1}{4\sqrt{\mu}}\sin(\sqrt{\mu}\tau), \quad S_6 = \frac{\lambda_1}{4\sqrt{\mu}}\cos(\sqrt{\mu}\tau)$$

$$\begin{cases} x_0(T_0, T_1) = a\cos(\sqrt{\mu}t + \theta) \\ v_0(T_0, T_1) = V\cos(\sqrt{\mu}t + \theta + \arctan\frac{\alpha}{\sqrt{\mu}}) \end{cases} \tag{17}$$

where the voltage amplitude V is given by:

$$V = \frac{\sqrt{\mu}}{\sqrt{\alpha^2+\mu}}a \tag{18}$$

The steady-state response of system (16), obtained by setting $\frac{da}{dt} = \frac{d\theta}{dt} = 0$, corresponds to a periodic solution of (5) and (6). Eliminating the phase, the amplitude a satisfies (19):

$$(S_1a + S_2a^3)^2 + (\frac{S_3}{2}a + \frac{S_4}{2}a^3)^2 = (S_5^2 + S_6^2)a^2 \tag{19}$$

The condition for (19) to have two real roots is given by:

$$(2S_1S_2 + \frac{S_3S_4}{2})^2 - 4(S_2^2 + \frac{S_4^2}{4})(S_1^2 + \frac{S_3^2}{4} - S_5^2 - S_6^2) > 0 \tag{20}$$

and the conditions for the stability of the steady-state response are given by:

$$2S_1S_2 + \frac{S_3S_4}{2} > 0 \tag{21}$$

$$S_1^2 + \frac{S_3^2}{4} - S_5^2 - S_6^2 > 0 \tag{22}$$

Note that for the undelayed system ($\lambda_1 = \tau = 0$), where ($S_5 = S_6 = 0$), the amplitude a satisfies (23):

$$a = \sqrt{-\frac{S_1}{S_2}} = \sqrt{-\frac{S_3}{S_4}} \tag{23}$$

Figure 2 shows the stability chart in the parameter plane (λ_1, τ) for $\omega = 2$ indicating the grey regions where stable quasi-periodic (SQP) solutions take place and the white region where the conditions (21) and (22) are satisfied, corresponding to stable periodic (SP) solutions. Figure 2(a) shows the stability chart in detail, while Figure 2(b) shown the time histories and the corresponding output power responses related to crosses labelled 1, 2, and 3 in Figure 2(a). From cross 1 (or 3) to cross 2, the response bifurcates from SP to SQP oscillations via a secondary Hopf bifurcation, producing a decrease of the amplitude response and the corresponding output power at cross 2.

The average power is obtained by integrating the dimensionless form of the instantaneous power $P(t) = \alpha v(t)^2$ over a period T, and is given by:

$$P_{av} = \frac{1}{T}\int_0^T \alpha v^2 dt \tag{24}$$

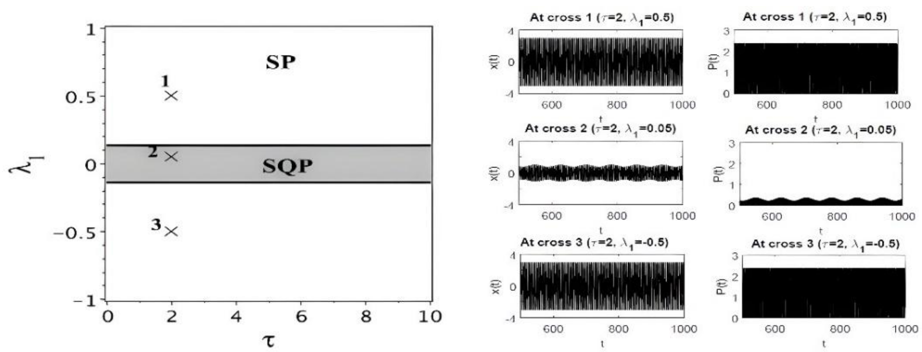
where $T = \frac{4\pi}{\omega}$. Hence, the average power expressed by $P_{av} = \frac{\alpha V^2}{2}$ takes the form:

$$P_{av} = \frac{1}{2}\left(\frac{\alpha\mu}{\alpha^2+\mu}\right)a^2 \tag{25}$$

where the amplitude a is given by (19) in the case of delayed system or by (23) in the case of undelayed one. Applying the maximization procedure, the maximum power is given by:

$$P_{max} = \frac{\alpha\mu a^2}{\alpha^2+\mu} \tag{26}$$

In (19), (23), and (26) will be used to study the effect of parameters on the steady-state response and on the output power of the harvester. Hereafter, we fix the parameters as $\mu = 1.175$, $\gamma = 0.024$, $\kappa = 10^{-2}$, $m = 6.83 \times 10^{-4}$, $\xi_m = 3 \times 10^{-3}$, $a_1 = 2.5$, and $a_3 = 130$.



(a) (b)

Figure 2. Stability behavior of the system in the (λ_1, τ) plane and the corresponding time and power responses; (a) stability map showing the SP and SQP regions and (b) time histories and harvested power responses for selected operating points, with $\omega = 2.1$, $U = 8$, $\mu = 1.175$, $\gamma = 0.024$, $\kappa = 10^{-2}$, $m = 6.83 \times 10^{-4}$, $\xi_m = 3 \times 10^{-3}$, $a_1 = 2.5$, and $a_3 = 130$

3. RESULTS AND DISCUSSION

Next, the influence of different parameters of the system on vibration and power amplitudes is examined. The observed enhancement of vibration amplitude and harvested power in the presence of delay modulation can be physically interpreted as a delay-induced parametric excitation mechanism. In this configuration, the time-delayed feedback term acts as an effective time-varying stiffness, whose modulation introduces an additional parametric energy input into the mechanical subsystem. When the modulation frequency approaches twice the natural frequency, a resonance-like amplification occurs, similar to classical parametric resonance phenomena.

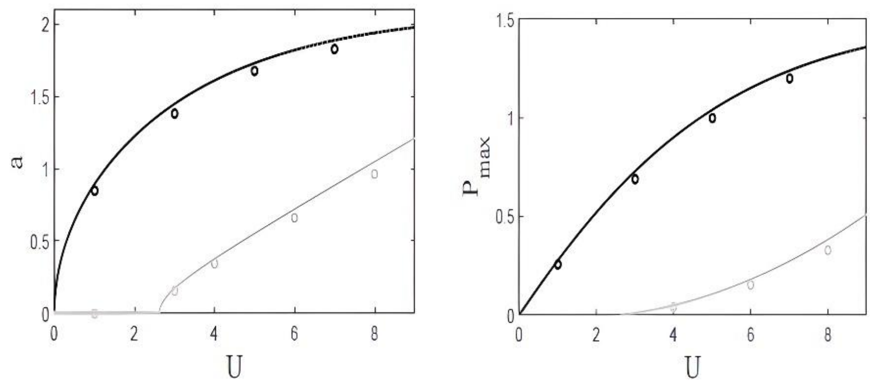
Furthermore, in the presence of galloping-induced aerodynamic forces, the delayed feedback modifies the phase relationship between the flow excitation and structural response. This phase shift enables more efficient energy pumping from the flow into the mechanical oscillator, thereby increasing the amplitude of oscillations. The nonlinear coupling between the galloping effect and the delay term also contributes to the redistribution of energy between stable and QP regimes, which explains the observed enhancement of output power in specific ranges of wind velocity. In (19) and (23) are used for periodic solutions for respectively delayed and undelayed system, and (26) is exploited to calculate the power response. The numerical simulation is conducted to support the analytical predictions by using dde23 algorithm [14].

In Figure 3 shown the variation of the amplitudes of periodic response (Figure 3(a)) as well as the maximum output power amplitudes (Figure 3(b)) versus the reduced wind speed U, for $\lambda_1 = \tau = 0$ (undelayed circuit, grey lines) and for $\lambda_1 = 0.3$, $\omega = 2.1$, and $\tau = 2\pi$ (delayed circuit, black lines). The analytical prediction is compared to numerical simulation (circles) obtained by using dde23 algorithm [14]. It can be observed from Figure 3 that the presence of a modulated delay amplitude in the mechanical component causes a significant increase of the vibrations and the output power in a certain range of U (Figure 3, black lines).

Numerical simulations are carried out using the dde23 solver [14] in MATLAB, which is suitable for solving delay differential equations. The system is integrated over a sufficiently long-time interval to ensure that transient effects vanish and steady-state responses are reached. The initial conditions are chosen as small perturbations around the equilibrium state to capture the natural evolution of the system dynamics. The history function required for the delay term is taken as a constant or a small-amplitude function over the delay interval. The parameter values used in the simulations correspond to those specified in the analytical study, allowing a direct comparison between analytical predictions and numerical results. The time step is automatically adjusted by the solver to ensure numerical accuracy.

From a practical point of view, the performance of the proposed EH system is sensitive to several physical and operational parameters. In particular, variations in piezoelectric coupling strength, mechanical damping, and electrical load resistance can significantly influence the harvested power. Moreover, environmental conditions such as wind fluctuations and structural uncertainties may affect the stability of the periodic response. These considerations highlight the importance of proper parameter tuning to ensure robust EH performance. In practical implementations, especially at micro-scale levels, material properties and fabrication tolerances may also play a crucial role in determining the overall system efficiency.

To further investigate the system behavior, the numerical analysis has been extended over wider parameter ranges, including variations in wind velocity, delay modulation amplitude, modulation frequency, and electrical damping coefficient. This extended parametric study provides a more complete picture of the nonlinear dynamics and EH performance. The additional results reveal transitions between different dynamical regimes and clarify the role of delay modulation in shaping both vibration amplitude and output power over a broad operating domain.

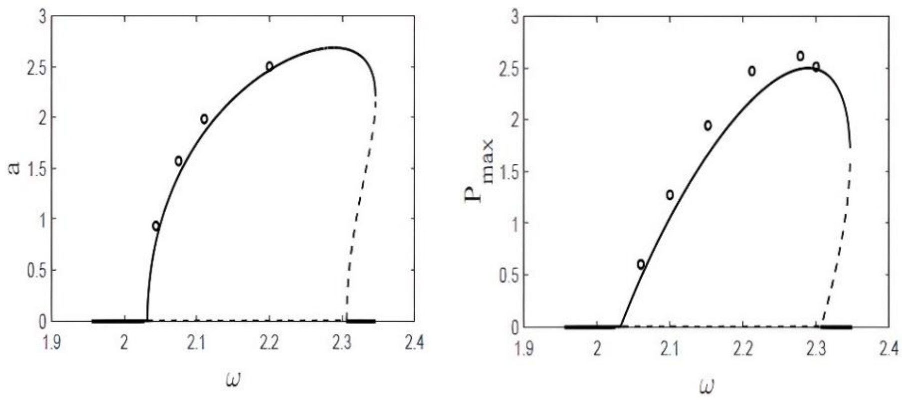


(a) (b)

Figure 3. Variation of steady-state vibration amplitude and maximum harvested power versus reduced wind speed U, where; (a) shows the vibration amplitude a and (b) presents the maximum harvested power P_{max}, with the black lines representing the delayed mechanical component ($\lambda_1 = 0.3$, $\omega = 2.1$, $\tau = 2\pi$) and the grey lines denoting the undelayed system ($\lambda_1 = \tau = 0$) [12]

The frequency response curves in term of amplitudes and power responses are illustrated in Figure 4 for the case where the time delay is present in the mechanical subsystem ($\lambda_1 = 0.3$, $\tau = 2\pi$). Figure 4(a) shows the variation of the vibration amplitude, whereas Figure 4(b) presents the corresponding maximum harvested power response. It can be observed from Figure 4 that the presence of a small delay amplitude in the mechanical has a significant effect on harvesting energy from periodic vibrations. The increase in harvested power observed under delay modulation can be physically interpreted as a delay-induced parametric excitation mechanism. The time-delayed feedback introduces an effective time-varying stiffness, which modifies the phase relationship between the aerodynamic galloping force and the structural response. When the modulation frequency approaches twice the natural frequency of the system, resonance-like amplification occurs, resulting in significantly larger oscillation amplitudes. From a system design perspective, this mechanism suggests that delay parameters can be used as an additional tuning tool to control resonance conditions and optimize energy transfer efficiency in flow-induced vibration harvesters.

The observed enhancement in vibration amplitude and harvested power can be attributed to a delay-induced parametric excitation mechanism. The modulated delay acts as an effective time-varying feedback stiffness that alters the phase relationship between the aerodynamic galloping force and the structural response. When the modulation frequency approaches twice the natural frequency, resonance-like amplification occurs, resulting in significantly increased energy transfer efficiency. From a design perspective, this suggests that delay parameters can be used as tunable control variables to optimize EH performance and to tailor the dynamic response of flow-induced vibration systems.



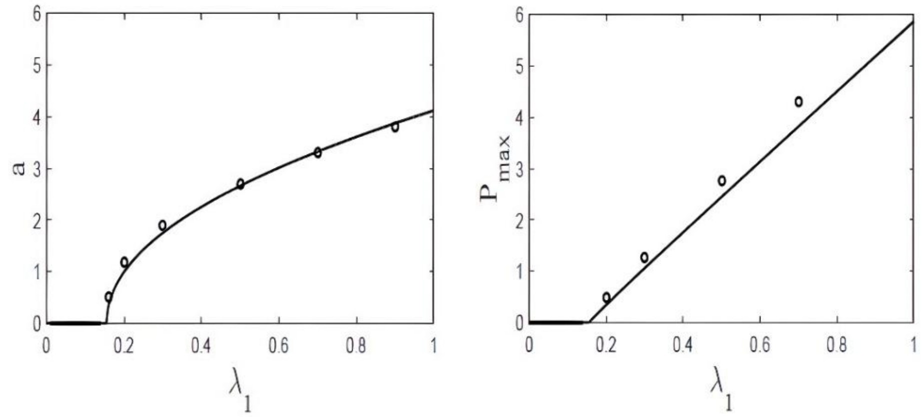
(a) (b)

Figure 4. Variation of the vibration amplitude and maximum harvested power versus ω for ($\lambda_1 = 0.3$, $\tau = 2\pi$); (a) shows the vibration amplitude a and (b) presents the maximum harvested power P_{max}

Figure 5 shows the variation of the amplitude of the responses (Figure 5(a)) and the powers (Figure 5(b)) versus the mechanical delay amplitude λ_1 for $\omega = 2.1$ and $\tau = 2\pi$. The analytical prediction is compared to numerical simulation (circles) obtained by using dde23 algorithm [14]. One can observe from Figure 5(b) that for small values of λ_1, energy cannot to be extracted from periodic vibrations. Beyond a certain value of λ_1, periodic solution appears, offering the possibility to scavenging energy with better performance.

The amplification observed in the system can be attributed to a delay-driven parametric injection of energy. In this context, the introduction of a modulated time delay alters the phase interaction between the aerodynamic excitation and the structural dynamics, effectively decreasing the apparent damping while promoting a more efficient transfer of energy. This phenomenon accounts for the resonance-like response detected at particular flow velocities, and underlines the importance of delay modulation as a tunable parameter for improving the performance of nonlinear EH systems.

A comparative analysis between configurations with and without delay clearly demonstrates the benefit of the proposed approach. Incorporating a time-varying delay significantly enhances the harvested power over an extended interval of flow velocities. Notably, both the maximum extracted power and the effective operating range are improved relative to the undelayed case. This enhancement is mainly due to a better phase synchronization between the galloping-induced aerodynamic forces and the structural motion, which facilitates a more efficient conversion of mechanical energy into electrical output through the piezoelectric coupling.



(a) (b)

Figure 5. Variation of the vibration amplitude and maximum harvested power versus λ_1 for ($\omega = 2.1$, $\tau = 2\pi$), where; (a) shows the vibration amplitude a and (b) presents the maximum harvested power P_{max}

Finally, Figure 6 illustrates the effect of the inverse electrical time constant, denoted by α, on the harvested power. For the selected set of parameters, there exists a range of α values within which the EH performance remains notably high.

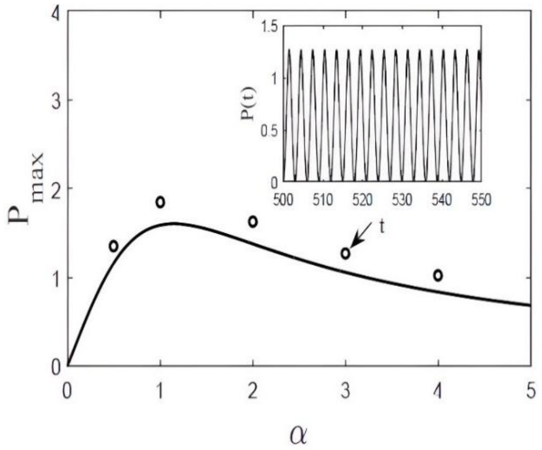


Figure 6. Power amplitudes vs. α for ($\omega = 2.1$, $\tau = 2\pi$, and $\lambda_1 = 0.3$)

From a system design perspective, the introduction of delay modulation provides an additional degree of freedom for controlling nonlinear dynamic responses in EH devices. By appropriately tuning the delay amplitude and modulation frequency, it is possible to shift stability boundaries and promote desirable high-energy periodic regimes. This capability is particularly relevant for practical EH devices, where adaptive control of system parameters can significantly enhance performance under variable flow conditions.

4. CONCLUSION

Periodic vibration-based EH has been extensively investigated in systems incorporating delayed nonlinear oscillators subjected to galloping-type aerodynamic excitation and coupled to an electrical circuit through a piezoelectric transduction mechanism. In the present configuration, the time delay is not constant but varies periodically; more specifically, the delay amplitude is modulated with a frequency close to the delay-induced parametric resonance. Such a modulation introduces an additional form of parametric excitation that can strongly influence the global dynamics of the oscillator and the efficiency of the EH process.

To analyze the system behavior, the method of multiple scales is employed, providing approximate analytical solutions that capture the steady-state vibration amplitudes as well as the corresponding harvested power. This asymptotic approach allows the derivation of modulation equations governing the slow evolution of vibration envelopes near the parametric resonance condition, offering valuable insight into the influence of time-delay modulation on the overall performance of the harvester.

The results reveal that when the modulation frequency of the delay amplitude approaches the delay parametric resonance, the system exhibits a substantial amplification of vibration amplitudes. This dynamic amplification, in turn, enhances the electromechanical energy conversion process, yielding a significant increase in the harvested power compared to the case without modulation. The enhanced response results from a constructive interaction between the delay-induced feedback and the periodic modulation, which effectively transfers energy from the aerodynamic flow to the electrical domain with higher efficiency.

These findings highlight the potential of exploiting time-periodic delay modulation as a passive control strategy for improving the performance of nonlinear vibration-based energy harvesters. They also suggest that such an approach could be applied in various aeroelastic or flow-induced vibration systems where time-delay effects naturally arise due to structural or feedback mechanisms.

Although the present study is mainly based on analytical development and numerical simulations, the lack of experimental validation is acknowledged. This is primarily due to the difficulty of experimentally realizing controlled time-delay modulation in nonlinear EH systems. Nevertheless, the numerical results obtained using the dde23 solver provide a high-confidence validation of the analytical predictions. Future work will focus on experimental implementation using programmable piezoelectric shunt circuits or active control systems capable of emulating delayed feedback, in order to further confirm the proposed theoretical findings.

The present study has certain limitations that should be acknowledged. The analysis is based on a weakly nonlinear approximation and assumes small modulation amplitudes of the delay term. Strongly nonlinear regimes, stochastic wind fluctuations, and multi-degree-of-freedom effects are not considered. In addition, the present model does not account for full experimental uncertainties or real-time control constraints. Future work will focus on experimental validation, robust control strategies, and multi-parameter optimization to improve the practical implementation of delay-modulated EH systems. Extensions to more complex structural configurations and realistic environmental conditions will also be investigated.

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